Separation Science and Technology
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713708471

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To cite this Article Free, Michael L. , Zhu, Joseph S. and Moudgil, Brij M.(1998) 'Use of a New Particle Contact Probability Filtration Rate Model to Determine the Effect of Particle Size Distribution in Filtration', Separation Science and Technology, 33: 1, 57 — 66

To link to this Article: DOI: 10.1080/01496399808544755

URL: http://dx.doi.org/10.1080/01496399808544755

SEPARATION SCIENCE AND TECHNOLOGY, 33(1), pp. 57–66, 1998

Use of a New Particle Contact Probability Filtration Rate Model to Determine the Effect of Particle Size Distribution in Filtration

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ABSTRACT

The Kozeny equation is commonly used to determine the rate of filtration. This equation is derived from the Poiseuille equation, which states that the rate of filtration has a fourth-order dependency upon the pore radius. The Kozeny equation uses the Poiseuille equation with an average pore size determined based upon porosity and particle surface area. Because the pore sizes vary widely depending upon the particle size distribution, the use of an average pore size can lead to significant errors in determining the rate of filtration for samples with large changes in size distribution and small changes in surface area. In this study the Poiseuille equation was used together with the pore size distribution, which was calculated based upon particle contact probabilities, in order to more accurately determine the effect of size distribution on the filtration rate. Artificial particle size distributions of silica beads and gypsum particles were used at relatively low pressures to demonstrate the usefulness of the proposed model.

INTRODUCTION

A considerable amount of work has been performed on filtration covering a wide variety of factors that affect the rate of filtration such as sedimentation, compressibility, porosity, etc. (1–7). In addition, some modeling of filtration has been performed based upon the Kozeny equation (8). However, these studies do not consider the effect of size distribution on filtration beyond what is already implicit in the commonly-applied Kozeny equation through the specific surface area term.

Most industrial filtration rates involving filter media that provide little resistance compared to the filter cake are often described using the Kozeny equation which is generally written per unit of available filter area as (1)

$$\frac{dV}{Adt} = \frac{\epsilon^3 \Delta P}{K\mu(1 - \epsilon)^2 S_p^2 L} \tag{1}$$

in which V is the filtrate volume, t is the filtration time, A is the filter area, ϵ is the porosity, ΔP is the change in pressure across the filter cake, K is a constant, μ is the viscosity, S_p is the specific surface area of the particles, and L is the thickness of the filter cake. The Kozeny equation is derived from the Poiseuille equation based upon the assumption that the rate of filtration can be modeled by considering the pores between particles as small cylinders. Using the tube analogy for a filter cake of unit length and unit surface area, the porosity is equal to the unit volume determined from the number of cylinders times the effective cross-sectional area per cylinder times the unit length. The Kozeny equation also assumes that the surface area of the tube is equal to the surface area of the particles times the solids volume fraction in the cake. In addition, the Kozeny equation assumes that an average cylinder diameter can be used to represent all of the pores present. Although the assumption of an average cylinder diameter does not lead to substantial errors in cases where the particle size distribution stays approximately the same, it leads to large discrepancies in cases where the particle size distribution changes significantly. The discrepancies are particularly acute when the specific surface area remains relatively constant and the size distribution changes significantly. This study accounts for the effect of pore size distribution using geometric and probabilistic calculations in order to determine the effect of size distribution on filtration based upon the Poiseuille equation and the size distribution data, rather than assuming an average pore size as the Kozeny equation does.

CONTACT PROBABILITY FILTRATION MODEL

For filtration of particles that are distributed among different size classes, it is useful to identify the distribution of pore sizes found in a given cross-sectional sample of filter cake normal to the direction of fluid flow. The following method was derived to analyze the effect of particle size and size distribution using spherical particles.

Consider three particles *i*, *j*, and *k* that are touching each other inside a filter cake normal to the direction of flow as shown in Fig. 1. The rate of filtration will be dependent upon the pore size between the three adjoining spherical particles *i*, *j*, and *k*. As the solution flows over the top of the particle, it must eventually make its way around the outer region, or equatorial region, of the particle before it can flow to the next level of particles. Here the equatorial region is the largest diameter of the spherical particle that is normal to the direction of flow. Because the flow must travel around the equatorial region of each of the particles, the three particle contact can be effectively modeled in two dimensions. By assuming that the pore between particles *i*, *j*, and *k* can be represented by a tube of equivalent cross-sectional area with an equivalent diameter, the flow of fluid through the pore can be described using the Poiseuille equation:

$$\frac{dV}{dt} = \frac{\Delta P \pi r^4}{8 \mu L} \tag{2}$$

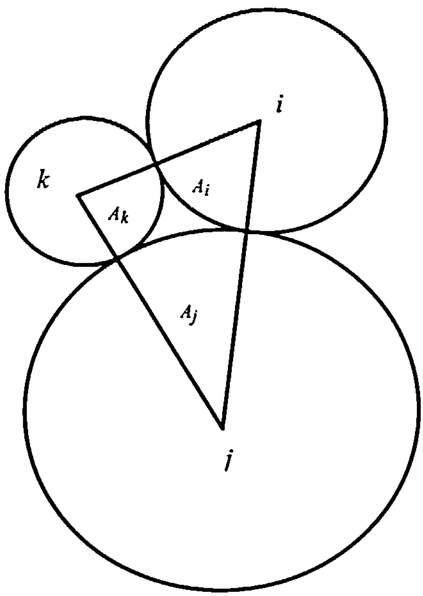

FIG. 1 Schematic two-dimensional diagram of particles *i*, *j*, and *k* contacting each other. This two-dimensional illustration represents a cross section of particles normal to the direction of fluid flow.

According to the Poiseuille equation, the rate of filtration will depend upon the effective pore diameter to the fourth power. In this study the cross-sectional area of the pore, which can be determined by subtracting the individual areas A_i, A_j, and A_k from the area of the triangle A_{ijk} (see Fig. 1), is used to determine an effective pore radius that can then be applied to the Poiseuille equation. The effective pore radius can be expressed as

$$r_{ijk} = \left[\frac{A_{ijk} - (A_i + A_j + A_k)}{\pi}\right]^{1/2} \tag{3}$$

Using the effective pore radius, the rate can be calculated for each pore size according to the probability of having a pore of that size. The probability of having a pore of a given size is dependent upon the probability of having a given set of particles contacting each other.

The number of different permutations of particles in contact with one another is given by (9)

$$C = N^S \tag{4}$$

where C is the number of permutations, N is the number of particle sizes, and S is the number of particles contacting each other, which is three in this analysis. For six particle sizes and three particles contacting each other, the number of permutations is 6^3 or 216. However, the probability of having particles i, j, and k together is given by invoking the multiplicative law of probability as (10, 11)

$$P_{ijk} = P_i P_j P_k \tag{5}$$

where

$$P_{(i)} = \frac{f_i 2\pi r_i}{\sum_{i=1}^{N} f_i 2\pi r_i} \tag{6}$$

and in Eq. (5), P_{ijk} is the probability of having particles i, j, and k contact each other, $P_{(i)}$ is the probability of having a particle of size i, f_i is the number fraction of the particles in size i. In other words, the contact probability is based upon the circumference of a given particle size times the number of those particles divided by the total (circumferences times number of particles) for all size classes; thus, a larger particle has a higher probability of contact than a small particle.

After determining the probability of contacts, the probability of achieving each possible pore size was determined using a spreadsheet containing each of the possible combinations. The rate of filtration through each pore size combination was then determined and weighted according to the probability of occurrence based upon the Poiseuille equation and the probability equations presented previously. The filtration rate based upon three particle contacts can be mathematically expressed as

$$\frac{dV}{dt} = \frac{\Delta P \pi}{8 \mu L} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} P_{ijk} r_{ijk}^4 \tag{7}$$

The filtration rate per unit area of filter surface is determined by dividing by the area of the triangle A_{ijk} for each pore. The resulting expression for the rate of filtration per unit area is given as

$$\frac{dV}{A dt} = \frac{\Delta P \pi}{8 \mu L} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{P_{ijk} r_{ijk}^4}{A_{ijk}} \tag{8}$$

The total probability for all 216 combinations for the six size classes used in this study was verified to be equal to 1.000.

EXPERIMENTAL PROCEDURES

Silica Sphere Filtration

All silica sphere filtration was performed using silica spheres obtained from Duke Scientific Company that were separated into size classes using standard Tyler mesh screens. The filtration was performed using a 400-mesh screen on top of a woven polymer filter cloth that was inserted into a Buchner funnel. The diameter of the filter was 3.85 cm. The viscosity of the phosphogypsum-saturated phosphoric acid solution was 2.9 cP as determined by a calibrated Ulbeholde capillary viscometer. The pressure differential was maintained at 7.4 psi by means of a valve and pressure regulator. The porosity of the silica sphere filtration cakes did not vary significantly between samples as indicated in Table 1. Also, the filter cake thickness remained the same as shown in Table 1; thus, in this study the

TABLE 1
Comparison of Cake Thickness and Cake Porosity for Silica Sphere Filtration Cakes

Size distribution	Number of observations	Cake thickness (cm)	Cake porosity
A	20	1.9 ± 0.1	0.44 ± 0.02
B	20	1.9 ± 0.1	0.45 ± 0.03
C	20	1.9 ± 0.1	0.45 ± 0.02
D	20	1.9 ± 0.1	0.44 ± 0.02
E	20	1.9 ± 0.1	0.45 ± 0.02

only parameter in the Kozeny equation that changed was the specific surface area that was dependent upon the particle size distribution.

RESULTS AND DISCUSSION

Model Evaluation Using Silica Spheres

The particle contact probability model was evaluated using silica purchased from Duke Scientific Corporation. Using spherical silica particles, several filtration tests were conducted using various particle size fractions in order to produce the size distributions presented in Table 2. The results from these tests along with the predicted results are presented in Fig. 2. The data in Fig. 2 show that the model gives more accurate predicted filtration rates than the Kozeny equation over an order of magnitude change in the rate of filtration for this ideal silica sphere system at relatively low pressures (<1 atm).

Model Evaluation Using Gypsum Crystals

Filtration experiments were performed using gypsum crystals formed as a by-product of phosphate fertilizer manufacturing. The gypsum crystals were used to determine the validity of the contact probability filtration rate model in an industrially relevant filtration scenario. The gypsum crystals were classified with respect to geometric size using standard sieves in the same way as silica spheres were, as discussed previously. Samples with narrow and broad size distributions were prepared by mixing the sized material in the proportions shown in Fig. 3. Filtration experiments using these size distributions showed that the filtration cake porosity changed by less than 2%. Additional experiments did not show evidence

TABLE 2
Silica Sphere Size Distributions[a]

Size distribution	84 μm wt%	63 μm wt%	49 μm wt%	41 μm wt%	35 μm wt%	25 μm wt%	14 μm wt%
A	15	15	15	15	15	15	10
B	0	5	25	40	25	5	0
C	0	0	0	100	0	0	0
D	0	100	0	0	0	0	0
E	100	0	0	0	0	0	0

[a] Note that the sizes given in the table represent the geometric mean particle size. In the model calculations for Distribution A, the 41 and 35 μm fractions were combined into one class of 38 μm to reduce the number of classes to six.

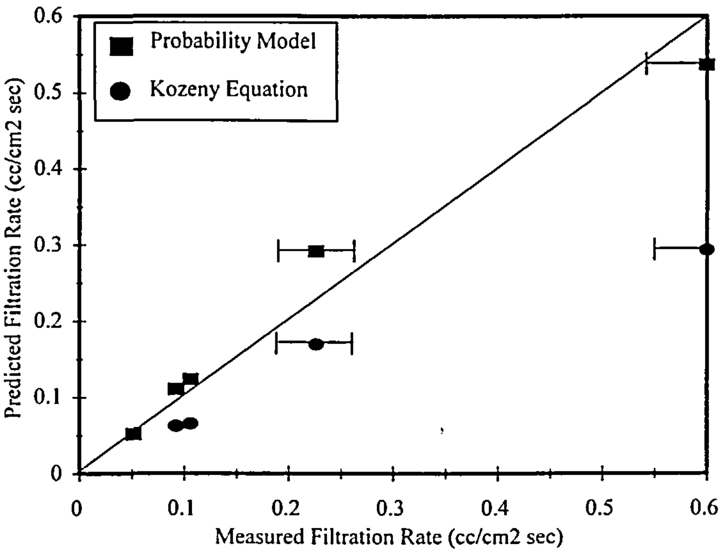

FIG. 2 Comparison of predicted and measured filtration rates for silica sphere filtration. The error bars represent the standard deviations.

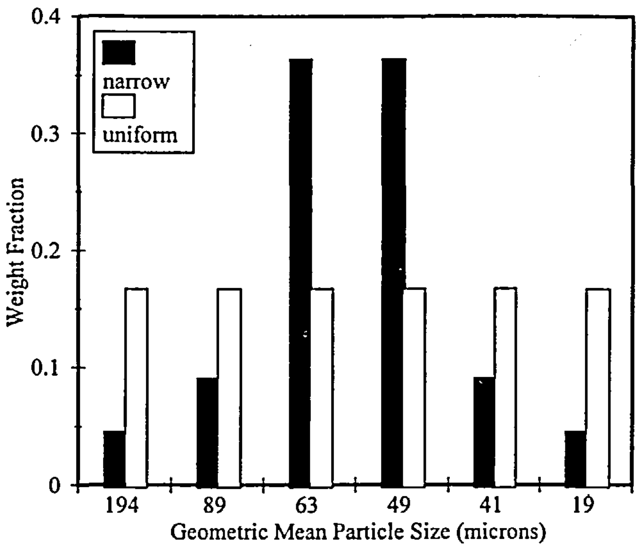

FIG. 3 Comparison of uniform and narrow size distributions of gypsum particles based upon the geometric mean particle sizes shown.

of cake compressibility during the filtration. The only parameter in the filtration rate equations that changed during the gypsum filtration experiments was the specific surface area. The measured filtration rates for the narrow and broad size distributions are presented in Table 3 along with the predicted rates based upon the Kozeny equation and the particle contact probability filtration rate model. Please note that the rate for the broad size distribution was used to determine the model and Kozeny equation constants. The Kozeny equation predicted a filtration rate increase of 32% from the broad distribution to the narrow distribution of gypsum particles. In contrast, the particle contact probability model predicted a 118% increase for the same change. The measured increase of 91% was much closer to the predicted values obtained using the particle contact probability model than the corresponding values obtained using the Kozeny equation, indicating that the proposed model more accurately predicts the effect of changes in size distribution than the Kozeny equation.

The observation that the model is more accurate than the Kozeny equation is not surprising given the fact that the Kozeny equation is derived based upon the assumption that all pores can be modeled as an array of equal diameter cylinders. As particle size distribution changes, the average cylinder diameter used in the Kozeny equation may not change, but clearly the distribution of pores will change based upon the probability of particle contacts. The proposed model takes these changes in pore size distribution into account, whereas the Kozeny equation relies primarily upon the particle surface area changes—that may or may not be closely related to the changes in pore distribution.

Comparison of the Kozeny Constant and Model Correction Factor

The Kozeny constant was calculated to be 18.6 for the silica sphere filtration tests. In contrast, the corresponding model correction factor was 5.4. The Kozeny constant is essentially an empirical correction factor that

TABLE 3
Comparison of Measured and Predicted Filtration Rates for the Uniform and Narrow Size Distributions of Gypsum Crystals Presented in Fig. 3

Size distribution	Kozeny equation rate ($cm^3/cm^2 \cdot s$)	Probability model rate ($cm^3/cm^2 \cdot s$)	Measured rate ($cm^3/cm^2 \cdot s$)
Uniform	0.22	0.22	0.22 ± 0.02
Narrow	0.29	0.48	0.42 ± 0.02

changes with a given set of particles. It is clear that the model correction factor of 5.4 is much closer to the ideal correction factor of 1.00, which would apply for flow through a perfect cylinder, than the Kozeny equation. Additional support for the particle contact probability model can be obtained by comparing the Kozeny constant and the model correction factor for the gypsum filtration tests. The Kozeny constant for these tests was 47.3, whereas the model correction factor was 7.1. The small increase in the model correction factor between the silica sphere and gypsum samples (5.4 to 7.1) compared to the rather large corresponding change for the Kozeny constant (18.6 to 47.3) adds additional support for the applicability and usefulness of the particle contact probability filtration rate model in predicting the effect of particle size distribution on the rate of filtration. It is clear, however, that the particle contact probability model does not take into account the effect of tortuosity, porosity, and extra surface drag on the fluid beyond what would occur in a tube, since a significant correction factor is required.

CONCLUSIONS

Results from this study show that the contact probability filtration rate model is more accurate in determining the rate of filtration than the conventional Kozeny equation in cases where the particle size distribution changes significantly. Analysis of filtration constants also indicates that the particle contact probability filtration model can be used for different systems without requiring large changes in the correction constant as is the case with the Kozeny equation.

ACKNOWLEDGMENTS

This research was funded by a grant from the Florida Institute of Phosphate Research (91-01-094R). The authors thank Andrew Garstkiewicz and Andrew Kulchin for their assistance in the laboratory. In addition, resources provided by the NSF Engineering Research Center for Particle Science and Technology at the University of Florida are gratefully acknowledged. Financial support of this research does not consititute endorsement of this publication by either of the funding agencies.

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Received by editor March 13, 1997
Revision received May 1997